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Lattice QCD: Going Beyond the Mass Spectrum

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The present status of lattice measurements with fermions is discussed. Methods are presented which improve the extraction of particle masses using staggered fermions. It is suggested that fermion physics on blocked lattices might show improvement as compared to that on lattices generated with the Wilson action. This idea is tested by comparing results from $18^3 \times 42$ lattices generated at $\beta = 6.2$ with results from the $6^3 \times 14$ lattices obtained from them by twice blocking with a $3^{1/2}$ transformation. Also, a comparison of $8^3 \times 16$ blocked and Wilson axis lattices is made. Finally, three point correlators are used to elucidate the properties of the scalar isoscalar channel.

KEY WORDS: Lattice gauge theory; staggered fermions; block spin transformation, scalar particles.

Monte-Carlo methods have considerably increased our understanding of pure gauge theories. This is less true, however, for gauge theories with fermions. A substantial amount of work has been done with fermions added in the approximation of ignoring fermion loops (the quenched approximation). This work has led to a zeroth order description of the low-lying hadron spectrum and to preliminary measurements of coupling constants and form factors. But there are many reasons to be dissatisfied with these results. For one thing, because the quenched approximation is uncontrolled, we do not know whether the results are really that much better than those obtained from less fundamental approaches such as the nonrelativistic quark model. Another problem is that if the pure gauge theory is anything to go by, it may be necessary to go to very large lattices (larger than most of those used to date) to enter the region where the perturbative

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renormalization group applies, so that a continuum limit can be taken. Finally, we would like lattice gauge theory to be a tool, giving us information that we have no other way to measure or calculate.

Thus lattice gauge theory (LGT) has now reached the stage where it can go beyond what I call the naive mass spectrum. In fact there has already been substantial progress toward finding a practical method of including fermion dynamics into LGT simulations.[1] This work is essential if LGT is to become a useful tool for phenomenology. It is equally important to find practical ways of measuring complicated quantities that we have no other way of obtaining, prime examples being the matrix elements of the weak interaction Hamiltonian, and the masses and couplings of exotic particles. We must also learn to deal with the fact that most particles decay when one includes fermion loops. Indeed, various attempts have already been made in this direction. Browner *et al.*,[2] Martinelli *et al.*,[3] and especially Bernard *et al.*[4] have proposed methods of evaluating weak interaction matrix elements and have made preliminary measurements. De Forcrand and collaborators[5] have finally pinned down the scalar glueball mass using source techniques. Mackenzie and Thacker[6] have studied the H dibaryon, a suspect in the Cygnus X-3 mystery, finding evidence that this exotic baryon is not stable. There are other examples. In this paper I present results of attempts in this direction obtained in collaboration with Phillipe de Forcrand, Rajan Gupta, Gerry Guralnik, Greg Kilcup, Apoorva Patel, and Tony Warnock.

Before proceeding it seems appropriate, given the nature of this conference, to speculate about the future of LGT. I have drawn up the following list of levels of possible progress

Hadron spectrum	
Form factors, coupling constants	Exotic particles: $GG, \bar{Q}QG$,...
Structure functions	Matrix elements, weak and superweak
	LGT with scalars *and* fermions
	LGT with chiral fermions

On the left-hand side are quantities which we can measure experimentally (I am assuming that the LGT in question is QCD). Lattice calculations of these quantities serve as an important check on the methods and, of course, on the correctness of the theory. On the right are the quantities and theories we would like to understand in order to build models at smaller scales than the QCD scale. As one moves down both lists the difficulty of the lattice measurement increases. *All* entries require the use of dynamical fermions if they are to give completely reliable results. Results obtained in the quenched approximation will give a guide to the answers. This is probably sufficient where there is at least an order of magnitude theoretical uncertainty or discrepancy as, for example, in the $\Delta I = \frac{1}{2}$ rule. In any case, for most applications involving fermions, the development of techniques does not require the use of dynamical fermions, since the inclusion of fermion dynamics only effects the set of gauge configurations to be used. For this reason I do not discuss dynamical fermions in the following.

The most interesting possible uses of LGT are, not suprisingly, the last two in the list. It is still unresolved whether the weak interactions are strong or weak! That is, it is not clear whether the weak interactions are described by the Weinberg–Salam Lagrangian with the gauge coupling being weak or strong. It is not even clear whether theories with fundamental scalars can have interacting continuum limits. To answer these questions we must simulate theories with fermions (preferably dynamical) *and* scalars.

Chiral theories—theories with fermions in complex representations of the gauge group—are perhaps the most interesting of all field theories. They have the potential of solving many phenomenological problems, e.g., they may have composite light fermions in their spectra (for recent work, see Ref. 7), yet we have very little reliable understanding of their dynamics. To put such theories on the lattice we must overcome the doubling problem (discussed by Kogut, Ref. 1) because of which lattice fermion representations are always real. An important first step has been made by Alvarez–Gaume and Della-Pietra[8] who relate the intrinsically chiral part of the effective action—the imaginary part of the fermion determinant—to a quantity calculable in a theory with real representations of fermions. A naive transcription of their continuum formula onto the lattice,[9] however, fails. This is a very interesting area for future study.

Having indulged in such speculations, I now return to what is possible today. Put another way, I must retreat from the title "Beyond the naive mass spectrum" to the more modest "ε beyond the naive mass spectrum."[10] I will describe our efforts to move down the list given above. At present our major goal is to measure pion and kaon matrix elements of weak and super-weak interaction Hamiltonians. Unfortunately, we have not yet reached this goal. Nevertheless, we have made steps forward, and the path is (more or less) clear.

More specifically, we wish to use staggered fermions to measure matrix elements. This is in distinction to Wilson fermions with which, to my knowledge, all calculations down the right-hand side of my list have been done. The merits of both types of fermion have been extensively discussed in the literature.[1] I will note only the following features. For Wilson fermions, the evaluation of quantities related to chiral symmetry breaking, e.g. $\langle \bar{\chi}\chi \rangle$ and $\pi - \pi$ scattering amplitudes, is difficult because there is no chiral symmetry that is even partially conserved. One has to make nonperturbative subtractions to regain the Ward Identities which follow from the chiral symmetry.[11] On the other hand, staggered fermions have a nonsinglet axial $U(1)$ symmetry which is only softly broken by mass terms, and which appears to be dynamically broken, just as in the continuum. This guarantees lattice versions of all the standard current algebra results, e.g., the relation of Gell-Mann, Oakes and Renner,[12] and the Weinberg scattering length formula,[13] involving no subtractions.[14] It can also be shown[14] that, for appropriately chosen operators, the chiral behavior of weak interaction matrix elements is as expected from current algebra and PCAC. But for most quantities subtractions are not necessary, and Wilson fermions are preferred. This is because the flavor and spin projections are straightforward for Wilson fermions, while for staggered fermions, as I shall discuss more below, they are complicated. A good example of this is the recent calculation using Wilson fermions of the mass of the H particle[6] (the lightest 6-quark state with flavor composition *udsuds*). The complicated spin contractions necessary to project onto the lightest state cannot easily be done with staggered fermions, as the required operators are nonlocal and involve fluctuating gauge links. In summary then, for processes involving pions, staggered fermions are preferred, and we will use them in the following.

The problems with staggered fermions arise from using 1-component objects to represent 4 flavors of 4-component spinors. The spin and flavor degrees of freedom are distributed over the 16 lattice sites of a hypercube. One problem this causes is that the lattice symmetry group includes shifts by a single lattice unit. This symmetry is only realized when the ensemble of gauge configurations is invariant under these shifts. A related problem is that the conserved vector and partially conserved axial currents are nonlocal. These currents must be used in the calculation of weak interaction matrix elements. But the matrix elements including these currents are subject to extra noise, because they involve fluctuating links. A further problem is that one cannot construct operators that project onto states of definite parity. This is because the 16 sites needed to construct fermion operators lie on two time slices. To combine them into an operator one needs to know the transfer matrix, i.e., one needs to solve the theory.

Things are not so bad in the pion channel. To understand this I must give some details of the staggered fermion symmetries.[3] These will also be useful below. Although the symmetry group only becomes *spin* $\times$ $SU(4)$ *flavor* in the continuum limit, it is convenient to use this group to desribe the states.[16] I will use gamma matrices to describe the flavor as well as the

[3] The symmetry group of staggered fermions has been studied recently by Golterman and Smit.[15] Kilcup and I have found a somewhat nicer way of looking at the group.[14]

spin, but denote the flavor matrices by T_5, etc. Then the pair of states with opposite parities are $\gamma_{spin} \times T_{flavor}$ and $\gamma_{spin}\gamma_5\gamma_0 \times T_{flavor}T_5T_0$. The pseudo-goldstone pion has quantum numbers $\gamma_5 \times T_5$, so its partner is $\gamma_0 \times T_0$. In the continuum limit the latter operator is the charge density of a conserved current, and so cannot create a massive scalar state. Thus of all the states the pion is most easily projected onto; away from the continuum limit the amplitude of the scalar partner should be small, and in practice it is in the noise. This is fortunate for studies involving pions, but it is not the end of the story of the scalar partner, as we shall see below. For the other spins both parities are present. The pairings are ρ/B, $\tilde{\rho}/A_1$, $\tilde{\pi}/\varepsilon$, where a $\tilde{\rho}$ means the state created by the spin operator $\gamma_i\gamma_0$, and $\tilde{\pi}$ is created by $\gamma_5\gamma_0$. The states are labeled according to their spin and flavor, the ε, for example, being the lattice-scalar, flavor-singlet state. Note that the lattice symmetry does guarantee that the three-spin components of the ρ have equal energy (though only at zero momentum), but it does not guarantee that $m_\pi = m_{\tilde{\pi}}$ or that $m_{\tilde{\rho}}$. These equalities should hold in the continuum limit, and the deviations from equality give an indication of the errors due to discretization.

A problem common to both types of fermion is that of needing very large lattices to reach the region where one can confidently take the continuum limit. This region of assymptotic scaling (AS) is that for which physical quantities scale according to the perturbative renormalization group (RG) equation, i.e., proportional to powers of e^{-1/g^2}. For pure gauge theories one needs to go to at least $\beta \equiv 6/g^2 = 6.1$ for this to be true, the nonperturbative β function differing from the two-loop assymptotic freedom prediction for smaller $\beta^{(17)}$. For $\beta \geqslant 6.1$ one expects lattice spacings such that

$$a < \frac{1}{2Gev} \approx \frac{1}{10} fm$$

so one wants $N_s > 10$ to reasonably accommodate hadrons, and of course $N_s < N_t$ to make the temperature small. We are using an $18^3 \times 42$ lattice at $\beta = 6.2$ and feel that this is the bare minimum. The problem then is that it is very time-consuming to do fermion physics on this lattice. One wants to solve the lattice Dirac equation on each lattice for a variety of quark masses and a variety of base points, and one wants to do this for as many lattices as possible. The inversions take hours of Cray time each, though, adding up to a considerable amount of computer time. Furthermore, to move down my list, one needs to develop and test new techniques, and for this a large lattice is inappropriate. Our attempted solution to this problem depends on the existence of a scaling region. In this region ratios of physical quantities are constant, but the physical quantities do not scale according to the perturbative RG. It is not clear that this notion is any more than a clutching at straws. If it does have some validity, however, then one should be able to find an improved action which has the same long distance properties as a lattice on the Wilson axis at $\beta > 6.1$, and yet has a much larger lattice spacing.

The obvious way to construct such an action for the gauge degrees of freedom is to use a real space RG transformation, blocking the original large lattice down to a smaller one. To the extent that the blocking transformation is good, the large Wilson loops on the small lattice will retain the information of those on the large lattice. Since fermion propagators are expansions in which the coefficients are Wilson loops, there is some reason to expect that the fermion physics calculated on the blocked lattices will also retain the long distance physics of propagators calculated on the large lattice. This is counterbalanced by the coarseness of the small lattice. The best way to procede is to improve the fermion action simultaneously, counteracting some of the effect of this coarseness. This can been done for Wilson fermions by integrating out the heavy short distance modes.[18] But this procedure fails for staggered fermions because the short distance modes are massless. They are precisely those modes which turn the one component fermions into 16-component fermions. One can nevertheless imagine attempts in this direction,[19] but here we simply use the standard staggered fermion action on the blocked lattice.

Thus we have calculated propagators on $6^3 \times 14$ lattices obtained by twice blocking the $18^3 \times 42$ lattices with the $3^{1/2}$ blocking transformation.[20] We have also used the same procedure to block the $24^3 \times 48$ lattices of Phillipe de Forcrand,[5] to $8^3 \times 16$ lattices. Calculations are quick and cheap on the small lattices, and we have used them to obtain results with good statistics and to develop and test techniques.

The data sample from which I will present results is as follows. The $18^3 \times 42$ lattices were generated with a 20-hit Metropolis algorithm. 400 thermalization sweeps were discarded, and then every 50th lattice was blocked to a $6^3 \times 14$ lattice. We have calculated propagators on 40 of these lattices. We are also in the process of calculating propagators on the $18^3 \times 42$ lattices every 250 sweeps, and at the time of writing we have results from 12–15 (depending on the quark mass) such lattices. These results are clearly important for comparison with the small lattices to test the idea of using an improved action, but our results on the large lattices are preliminary. We also have calculated propagators on 25 of the blocked $8^3 \times 16$ lattices.[5] For purposes of comparison we have also generated 29 (1 was lost!) $8^3 \times 16$ lattices at $\beta = 5.6$ on the Wilson axis. Our inversions are done with a standard even-odd partitioned conjugent gradient routine.[21] The code has to be optimized depending on the machine being used. Our time sliced $18^3 \times 42$ program completes 500 iterations (each iteration involves two multiplies by the covariant derivative) on three colors in 50 minutes on a CRAY-XMP, if i/o is done with a solid-state disk. One thousand iterations are required for convergence at the smallest quark masses. The smaller lattices can be run entirely in memory, and our inverter does 500 iterations in about 70 s on the $8^3 \times 16$ lattices. Thus, the in-memory inverter is a little faster. The propagators were calculated with antiperiodic boundary conditions in space and both antiperiodic and Dirichlet boundary conditions in time.

In what follows I concentrate on the parts of our results that are most relevant to going beyond the naive mass spectrum. More details of the results from the $6^3 \times 14$ lattices can be found elsewhere.[10] The $8^3 \times 16$ and $18^3 \times 42$ results will be written up fully elsewhere.[22]

The first results I will present are for Wilson loops calculated on the blocked lattices. We would like to calculate the string tension on all the lattices and compare their values. Unfortunately, we do not yet have enough statistics. What we can do is see how the Wilson loop values compare to those on the Wilson axis. This gives us some insight into how our improved action differs from the Wilson action. We find, for both $6^3 \times 14$ and $8^3 \times 16$ lattices, that as the loop size increases the loop values converge to Wilson axis values for $\beta \approx 5.47$ and 5.57, respectively. These values are consistent with expectations from Monte Carlo RG measurements of the nonperturbative β function.[17] Smaller loops, however, correspond to smaller values of β. Thus at short distance the gauge degrees of freedom fluctuate more with our blocked action than at a point on the Wilson axis where long distance properties match. At the shortest distances this is probably an artifact of the RG transformation. At intermediate distances, however, this trend must represent the attempt of the blocked action to improve over the Wilson action. It certainly means that if we could extract a string tension it would be different from that at the corresponding point on the Wilson axis. Thus the physics of the blocked lattice is different from that of the Wilson axis, but we cannot tell whether it is better or worse. In one respect, though, it is definitely worse. The larger fluctuations of the gauge links will increase the noise in the correlators with nonlocal operators which I discuss below.

I next turn to the mass spectrum of the theories. We can test whether or not using blocked lattices improves the fermion physics in two ways: comparison with Wilson axis results and comparison with results on the large parent lattice. (I will use the term "large" to refer to the $18^3 \times 42$ lattice from now on, the other lattices being "small".) To make comparisons we need at least two physical quantities. For the pure gauge sector, we cannot measure the string tension from our small sample, and we certainly cannot measure the glueball mass. This leaves the mass spectrum, and I use three quantities[4]—f_π, m_ρ, m_N. I have given them in order of decreasing reliability of measurement; all other quantities are measured too unreliably to be useful. Even though f_π is better measured, there is an uncertainty in how to relate it to the continuum value, because there are finite perturbative corrections to this relation which can only be calculated for lattices in the AS region. In the AS region, these corrections are as large as 30%.[(24)]

There are problems in the extraction of masses. To do this convincingly one has to see a clean exponential signal for a number of time steps. For staggered fermions one has to disentangle two such exponentials, one of which is alternating. One has to go to large enough times to remove radial excitations and hope that by then the signal has not disappeared in the noise. Fitting is very sensitive to the assignment of errors, since a point on the tail of the exponential with abnormally small error can have a large effect on the fit. As has been pointed out by Bowler et al.[(21)(23)] this sensitivity has led to widely differing results being obtained from similar data. It makes fitting an art rather than a science, and one should be wary of the results and errors quoted by anyone, myself included. Our fitting has become progressively more sophisticated with time, and consequently the systematic errors decrease as one moves from the results obtained from our $6^3 \times 14$ lattices, to the $8^3 \times 16$, and finally the $18^3 \times 42$. For the last case we do fits with and without symmetrization, and search for stability when we remove various numbers of points both near and far from the base.

I give in Table I the results for f_π, m_ρ, and m_N extrapolated to $m_q = 0$. Zero mass is essentially the same as the physical up and down quark

[4] Bowler et al.[(23)] have a nice method of comparing data in which they plot m_N/m_ρ against m_π/m_ρ and compare the curve to a theoretically motivated one. Here, however, this method adds little insight.

Table I. Results in Lattice Units[a]

Quantities (MeV)	$6^3 \times 14$	$18^3 \times 42$	$8^3 \times 16$(blocked)	$8^3 \times 16$(Wilson)
$f_\pi(93)$	0.20(01)	0.03(01)	0.15(01)	0.18(01)
$m_\rho(776)$	1.45(01)	0.47(07)	1.10(20)	1.10(20)
$m_N(938)$	1.80(40)	0.45(15)	1.90(30)	1.90(30)

[a] All numbers are extrapolated to $m_q = 0$. Errors are statistical. Results from the $18^3 \times 42$ lattice are preliminary. Experimental numbers are given for comparison.

masses, and I give the experimental numbers for comparison. I stress that the $18^3 \times 42$ results are preliminary. The errors quoted are statistical and they are somewhat uncertain since they involve an estimate of the error introduced by the extrapolation. The systematic errors have been discussed above. The most reliable conclusion that can be drawn from this table is that it is very hard to say anything! The errors in the masses, particularly m_N, are very large. The errors in f_π are smaller, but then there are the systematic errors discussed above. One can procede blindly and say that the $6^3 \times 14$ blocked lattice results appear better than the $8^3 \times 16$ Wilson axis results, but then the $8^3 \times 16$ blocked results are only marginally better ($f_\pi/m\rho$ is smaller) than those on the $8^3 \times 16$ Wilson lattice, so this is not much of a victory for the blocked lattices. The $6^3 \times 14$ and $18^3 \times 42$ results should differ by a factor of 3 if long-distance physics is retained by the RG transformation, and this is clearly possible for m_ρ and m_N, but not for f_π. However, the corrections in the relation between lattice and continuum f_π values need not be the same, and the $18^3 \times 42$ data is preliminary and has large errors, so this is not conclusive.

Thus I find it hard to draw any quantitative conclusions from these comparisons. However, I think some qualitative conclusions can be drawn. If I take the $8^3 \times 16$ results as typical of blocked lattices, as I am inclined to do since the fitting is more reliable than on the $6^3 \times 14$ lattices, then there really does seem to be an improvement in m_N/m_ρ when one goes to the large lattice. This is expected, but it is gratifying to see it anyway. Also there is a significant improvement in the recovery of $SU(4)$-flavor symmetry on the large lattices. This is most striking for the π and $\tilde{\pi}$: for the smallest quark masses on the small lattices the $\tilde{\pi}$ is nearly as heavy as the ρ and shows no signs of goldstone behavior. On the large lattice we find degeneracy! A similar, though less striking result holds for the ρ and $\tilde{\rho}$. There are also some clear differences between the results on the $8^3 \times 16$ blocked and Wilson axis lattices. First, the signals of particles not discussed above, e.g., the A_1, are clearly stronger on the blocked lattices. Second, the ε mass, for small quark mass, is close to twice the pion mass on the blocked lattices, whereas on the Wilson axis it is heavier, almost as heavy as the ρ. (We have no results for the ε on the large lattices at present.) I will return to this difference below.

Summarizing, I would say that the idea of using blocked lattices to obtain improved results is not very useful. Simultaneous improvements in the fermion action might help. But even if an improvement scheme can be found for staggered fermions, the uncertainties in fitting seem to me to nullify any probable gain. Furthermore, the proximity of the small lattices to the strong coupling region causes poor results for those mesons which are infinitely heavy in strong coupling. I shall discuss an explicit case of this below. The same is not necessarily true for Wilson fermions, and reasonable results have in fact been obtained.[18]

I now turn to the other function of the small lattices: to serve as a testing ground for new techniques. In fact we have used two new techniques in obtaining the results given above. The first is used in the evaluation of f_π. We use the relationship

$$f_\pi^2 = \left(\frac{m_q}{m_\pi^2}\right)\left(\frac{\bar{\chi}\chi}{2}\right)$$

where the left-hand side is to be evaluated at $m_q = 0$. This requires the evaluation of $\bar{\chi}\chi(m_q = 0)$. To do the extrapolation we used[14]

$$\left(1 - m_q \frac{\partial}{\partial m_q}\right) \bar{\chi}\chi(m_q) = 2m_q \sum_{n_{even}} |G(n; 0)|^2$$

where $G(f; i)$ is the quark propagator from i to f. The left-hand side is the intercept of $\bar{\chi}\chi$ at $m_q = 0$, calculated using linear extrapolation. The right-hand side can be evaluated with only a single propagator calculation. This method reduces the number of values of m_q needed to estimate $\bar{\chi}\chi(m_q = 0)$. For the small lattices it is helpful, but its real strength is on the large lattices where $\bar{\chi}\chi$ is a rapidly increasing function of m_q. The extrapolated value for the large lattice quoted in Table I would have been very hard to obtain without using this method, because at smallest mass $m_q = .01$, $\bar{\chi}\chi$ is .061(2).

The second technique is more important. As explained above, apart from the pion channel, all mesonic channels have contributions from both positive and negative parity states. However, if we look in the continuum at 2-point correlators in which one operator has an extra γ_0 in its spin projection compared to the other operator, then only the ρ and π can be both created and destroyed. All the positive parity states couple to only one of the operators. On the lattice the equivalent to adding a γ_0 is adding a time-directed link. However, on the lattice we can no longer make a rigorous argument excluding positive parity states. We can only appeal to the continuum limit. We have calculated correlators between local operators and operators nonlocal in time,[10] and find, somewhat to our suprise, that all positive parity signals are in the noise. Thus we can fit with a single exponential. For the π this makes no difference. In fact the local–nonlocal correlators in the π channel are related by an identity to the local–local correlators,[14] so no new information is gained. But the m_ρ, $m_{\tilde{\rho}}$, and $m_{\tilde{\pi}}$ results are significantly improved, particularly on the large lattice. Again this technique requires no extra propagator calculations and should be used in conjunction with 2-point correlators involving only local operators, to extract more reliable masses for the states of both parities.

Finally, I turn to our attempts to go beyond the mass spectrum. Our first attempt is related to the need to use nonlocal vector and axial operators to measure matrix elements. We take a first step by measuring 2-point correlators with these operators to see whether the fluctuating gauge links destroy the signal. 3-point functions using the vector operator, e.g., the pion form factor, have been measured for staggered fermions in $SU(2)$.[25] For these, however, one can show that at zero momentum the equations of motion guarantee that the gauge link fluctuations do not effect the signal.[14] This is not true for the 2-point correlators in which both operators are nonlocal in space. To calculate these correlators one needs propagators from two adjacent base points. We have done this calculation on the blocked $6^3 \times 14$ and $8^3 \times 16$ lattices. The channels for these operators are the same as for local operators, but there is an additional ρ/A_1 channel and also another $\tilde{\rho}/B$ channel. We find that, indeed, the signal is noisier that those in correlators with local operators. There are signals, nevertheless, which for most m_q are in the π, the ε, one of the ρ, and both of the $\tilde{\rho}$ channels. The masses of the three ρs and the ε are comparable to those of their local partners. On the other hand, the nonlocal π is heavier than the local (goldstone) π, behaving similarly to the local $\tilde{\pi}$. There is evidence of positive parity particles, but it is too weak to fit. The most striking result, however, is that the channels with the poorest signals are those corresponding to the vector and axial currents. This is disappointing, but it is perhaps not suprising as, in strong coupling, the states in these two channels are infinitely heavy.[5] Clearly, more study is needed. In particular, it would be interesting to compare the blocked and Wilson axis $8^3 \times 16$ lattices since the signal on the Wilson axis lattice may be better as the gauge links fluctuate less. The results from our large lattices should be better still.

This brings me to my final subject, the epsilon: the lattice scalar state which corresponds to the scalar, isoscalar in the continuum limit. Studying this particle turns out to be very instructive for a number of reasons. First, the techniques needed are also required in matrix element computations. Second, the experimental situation in the scalar, isoscalar channel is confused, and LGT can actually be used as a helpful tool. And finally it turns out that the problem of decaying particles that will be faced with dynamical fermions has to be faced here. Thus indeed we are inching ε beyond the mass spectrum.

[5] I thank Apoorva Patel for enlightening me on this point.

I only briefly discuss the technique, since it is a variation on the source technique of Bernard et al.[26] For matrix elements, one needs to measure 3-point functions in which the propagators do not share a common endpoint. To project onto a momentum for two of the three points of the correlator requires propagators with bases at every point on at least one time slice. This is clearly too time-consuming. Instead we first calculate a propagator with a δ function source at the base point. Then we use one time-slice of this propagator, weighted with phases, as a source for a second propagator. Finally we contract the first and second propagators. Thus for each momentum, and each time-slice of the pion source, one has to calculate only one extra propagator.

We use this technique to calculate the $\varepsilon-\pi-\pi$ 3-point function. Recall that we left the ε as a particle with $m_\varepsilon \approx 2m_\pi$ on the blocked lattices and with $m_\varepsilon \leqslant m_\rho$ on the Wilson axis. Other lattice calculations have also found a light ε, but its mass appears to increase with increasing β,[27][28] such that at $\beta = 6.0$ on the Wilson axis it is heavier than the nucleon.

Experimentally, $\pi-\pi$ scattering and K decays[29] rule out that $m_\varepsilon \leqslant 2m_\pi$. The lightest candidate state in the scalar isoscalar channel is the $S^*(980)$, but this most likely has flavor composition $us\bar{u}\bar{s} + ds\bar{d}\bar{s}$.[30] It can be interpreted either as a $K\bar{K}$ bound state[31] or as a bag exotic[30]. The next state in the channel is the very broad $\varepsilon(1300)$, which appears to be made primarily of up and down quarks. The lattice ε is made of light quarks, and so one should consider the partner of the S^* in which the s quarks are replaced by u and d quarks. In the bag model[30] this state is extremely unstable, lying well above the 2-pion threshold. Jaffe and Low[32] analyze the effect of this unstable state using the P matrix and find that it can account for the broad region of attraction in $\pi-\pi$ scattering [the old $\varepsilon(600)$[33]] that lies below the S^*. In the bound-state picture,[31] however, there is only a weak attraction between the pions, which has little effect on the phase shift. In this view the $\varepsilon(600)$ is only the tail of the very broad $\varepsilon(1300)$.[34][35]

Thus the theoretical and experimental situations in this channel below the S^* are both confused. We have tried to shed some light by calculating $\varepsilon-\pi-\pi$ 3-point functions. Doing this also allows us to test our source technique. We have only done this calculation on the $6^3 \times 14$ lattice, and with all particles at zero momentum.

Depending on the time-ordering of the three operators, different physical processes are allowed. If the two pions lie on the same side of the ε, then either a resonant ε is being created, which propagates and then couples to two pions, or the scalar operator is creating two pions in a nonresonant fashion. A combination of resonant and nonresonant "ε" is also possible. If the ε lies between the pions, one is measuring the matrix element $\langle \pi | \bar{\chi}\chi | \pi \rangle$. This is not all, however. If one of the pions lies between the other π and the ε, the positive parity part of the π operator can have a matrix element between the π and the $\tilde{\pi}$ (the latter created by the ε operator): $\langle \tilde{\pi} | 0^+ | \pi \rangle$. In the continuum limit, this is a flavor non singlet charge and its value should be 1. It is interesting to know the lattice value, but for present purposes it a nuisance.

All these processes are included in our fits to the data. We find that it is *not* possible to obtain good fits for small quark mass unless there is a resonant ε contribution, with the mass of the ε increased significantly over its 2-point correlator value. The numbers are given in Table II. The 3-point functions require an ε almost as heavy as the ρ. Table II also shows the reduced width of the ε, calculated from the coupling constant measured from the 3-point function

$$\Gamma^{red} = \frac{\Gamma(\varepsilon \to \pi\pi)}{\sqrt{1 - 4m_\pi^2/m_\varepsilon^2}} = \frac{3g_{\varepsilon\pi\pi}^2}{8\pi m_\varepsilon}$$

The table shows that the ε width can exceed half of its mass. Thus, the 3-point functions want a resonant ε with a mass definitely $2m_\pi$, and with a very strong coupling to two pions.

This result is in apparent contradiction with that obtained from the 2-point functions. We have, however, found a simple explanation for this: on the blocked lattices we are indeed seeing two zero momentum pions in the ε channel, *not* a resonant state. This is possible even though we are working in the quenched approximation, because the symmetry which forbids the ε to couple to two pions is not an identity. It is only good after averaging over gauge fields, and thus the effect is only statistically zero. One can make two pions with only a quark and an anti-quark line by sending either one in a loop from the base point to the final point, then back, and then

Table II. Results from the Blocked $6^3 \times 14$ Lattices[a]

m_q	$2m_\pi$	$m_\varepsilon(2pt)$	$m_\varepsilon(3pt)$	m_ρ	$\Gamma^{red}/m_\varepsilon$
.030	0.96(8)	0.94(5)	1.31(3)	1.39(8)	.78
.045	1.16(4)	1.12(5)	1.35(2)	1.39(5)	.66
.060	1.32(4)	1.24(4)	1.41(2)	1.41(3)	.58
.100	1.64(4)	1.46(4)	1.54(2)	1.47(2)	.44
.250	2.42(2)	1.83(6)	2.02(2)	1.68(1)	.38

[a] All numbers are in lattice units. Errors are statistical. 2 and 3 pt refer, respectively, to results from 2- and 3-pt correlators. Γ^{red} is defined in the text.

back again, before it is contracted with the final operator. For this to work, one of the pions must annihilate into gluons.[6] The staggered fermion discrete flavor symmetry, which forbids this, is a shift symmetry. This is fully realized only if the ensemble of gauge configurations shares this symmetry, which it will not do in a finite sample. If this explanation is correct the effect should decrease as the square root of the number of configurations used. We hope to check this in the future.

This also explains the difference between the blocked and Wilson axis results, and the increase of m_ε with increasing β. The point is that the blocked lattices have links which fluctuate more than their Wilson axis counterparts, so that the shift symmetry will be less well realized for a fixed number of configurations. Similarly, the symmetry will become better realized as one moves to larger β along the Wilson axis. For infinite β the symmetry is exact. I am thus claiming that looking at 2-point functions one sees, *with a naive fit*, more of a resonant state and less of the spurious 2π state as one moves to larger β on the Wilson axis. It is not clear, however, what the true value of m_ε is. If we accept the result obtained at $\beta = 6.0$[28] then it may be almost as large as m_{A_1}.

But, whatever the true mass is, we should be able to extract it at any β from our 2- and 3-point correlator data. Notice that, if our explanation is correct, the situation we face is similar to that which will be faced when using dynamical fermions for all particles except the N and π, somewhat ameliorated by the fact that our decay vertex is suppressed. We have refit our 2-point correlator data with a form which includes ε propagation with a single decay and recombination, one of which is allowed, the other being suppressed by the shift symmetry. The decay and recombination can occur at any times as long as they are ordered. We only include zero momentum pions in the intermediate state, since the energy of a pion with the smallest allowed momentum is large on our lattice. Thus the fitting function is an approximate lattice version of a one-loop bubble diagram. It has terms proportional to $e^{-m_\varepsilon t}$, $te^{-m_\varepsilon t}$, and, most importantly, $e^{-2m_\pi t}$. Our data can be fit with such a form with m_ε anywhere in the range 1.3–1.8 (in lattice units), for the smaller quark masses. If we use the allowed vertex found in 3-point correlator fits, we find that the suppressed $\varepsilon - \pi - \pi$ vertex is two orders of magnitude smaller than the allowed vertex. This supports our explanation. But there are not enough points to thoroughly test the fitting function, so no definitive conclusions can be drawn.

To be consistent we must include the effect of decays into the 3-point functions. The decay (and subsequent recombination) can occur prior to

[6] Notice that this type of diagram is not forbidden if the pions are replaced by ηs, but on our lattice these states are heavy and their contribution is suppressed.

the final coupling to two pions. We expect this to be a smaller effect for the 3-point function, because the ε propagates for a shorter time. Nevertheless, it means that our estimate of the m_ε from the 3-point correlator will be systematically low.

What do I conclude from all of this? I can see two possibilities. The first is, when a simultaneous fit to both 2- and 3-point correlators is done, it will reveal an ε considerably heavier than the ρ, perhaps heavier than the 1.06 GeV quoted at $\beta = 6.0$.(28) Such a state is very similar to the quark model $\bar{q}q$ scalar. Note that neither lattice nor quark model calculations include the effect of decays, so they can be compared. The 3-point correlator data will still give this particle a strong coupling to two pions. Thus the inclusion of quark loops could significantly shift the mass. This is exactly what Tornqvist claims,(34) with the state ending up as the $\varepsilon(1300)$.

The second possibility is that the ε is truly light with $m_\varepsilon \approx m_\rho$. This requires rejecting the result at $\beta = 6.0$.(28) It is the view that we have advocated.(10) The state would be very broad and would be identified with the "$\varepsilon(600)$." I think that this possibility has some chance of being correct. By performing the fits discussed above we have supposedly isolated a state which, in the quenched approximation, cannot decay into two pions. However it still can have a $\overline{\bar{q}q}qq$ intermediate state. The extra attractive energy which Jaffe(30) found in the bag model could be present here. Thus the state could be one of Jaffe's primitives, a state which disappears when the decay channels are opened up. This is a different state from that found in the quark model, and so there is no contradiction with the quark model. Presumably the $\bar{q}q$ state would be an excitation, hiding in the data.

To distinguish between these two possibilities more measurements are needed. We need simultaneous fits of 2- and 3-point correlators. Also 4-point correlators can be calculated and added to the analysis. And, of course, results from large lattices can easily discriminate between the possibilities. We are working on these things.[7]

In conclusion, even though I have not put a great deal of meat on the bones of "going beyond the mass spectrum," I am pleased that the lattice appears to be able to help resolve phenomenological problems. It is encouraging that it is possible to extract numbers in fairly complicated processes using staggered fermions. We have in fact extended our 3-point correlator analysis to the A_1 and nucleon channels, with successful results,

[7] I have not discussed the scalar glueball, which is expected to have a mass between 0.5 and 2.0 GeV, and confuse this channel further. Essentially, I have assumed that it is heavier than the S^* and plays no important role. If it is lighter it could usurp the role of the ε in the second possibility discussed in the text. It is hard, however, to accommodate two light scalar states in the data.(36)

and we are looking at 4-point correlators. But it seems unlikely that using blocked lattices with staggered fermions is a useful substitute for working on large lattices, at least for extracting credicle physical numbers. We are, however, continuing to use the $8^3 \times 16$ lattices to develop the programs needed to extract matrix elements.

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